

# Magnetic Quantum Oscillations of the Conductivity in Two-dimensional Conductors with Localization

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An analytic theory is developed for the diagonal conductivity  $\sigma_{xx}$  of a 2D conductor which takes account of the localized states in the broaden Landau levels. In the low-field region  $\sigma_{xx}$  display the Shubnikov-de Haas oscillations which in the limit  $\Omega\tau \gg 1$  transforms into the sharp peaks ( $\Omega$  is the cyclotron frequency,  $\tau$  is the electron scattering time). Between the peaks  $\sigma_{xx} \rightarrow 0$ . With the decrease of temperature,  $T$ , the peaks in  $\sigma_{xx}$  display first a thermal activation behavior  $\sigma_{xx} \propto \exp(-\Delta/T)$ , which then crosses over into the variable-range-hopping regime at lower temperatures with  $\sigma_{xx} \propto 1/T \exp(-\sqrt{T_0/T})$  (the prefactor  $1/T$  is absent in the conductance).

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Despite more than two decades of intensive studies, some open questions remain in the quantum magnetic oscillations of the 2D conductors. Even for the most studied case of the integer quantum Hall effect (IQHE) a coherent description is absent for different field and temperature regimes observed in the diagonal conductivity  $\sigma_{xx}$  [1]. In particular, it is not clear so far why quantum oscillations in  $\sigma_{xx}$  survive in spite of that most states within the broaden Landau levels (LL) are localized (i); Why  $\sigma_{xx} \rightarrow 0$  between the peaks in the limit  $\Omega\tau \gg 1$ , if at low fields it displays a standard Shubnikov-de Haas (SdH) oscillations (ii); Why with the decrease of temperature,  $T$ , the peaks in  $\sigma_{xx}$  display first a thermal activation behavior  $\sigma_{xx} \propto \exp(-\Delta/T)$ , which then crosses over into the variable-range-hopping (VRH) regime at low temperatures with  $\sigma_{xx} \propto 1/T \exp(-\sqrt{T_0/T})$  (iii); Why the prefactor  $1/T$  is absent in the conductance (iv).

The localization in the IQHE picture plays a crucial role. It is believed that extended states are at the center of the broaden LL and all the other states are localized [1]. At high fields the localized states mean that Landau orbits drift along the closed equipotential contours of the impurity potential. At places where contours come close electrons can tunnel from one contour to another providing thereby a conductivity mechanism through the extended states. The diagonal conductivity  $\sigma_{xx}$  and the Hall conductivity  $\sigma_{xy}$  are closely related in the IQHE. The peaks in the  $\sigma_{xx}$  are exactly at the same fields where  $\sigma_{xy}$  transits from one plateau to another. The ideal picture of the IQHE at  $T = 0$  assumes that  $\sigma_{xx} = 0$  within the plateaus while the  $\sigma_{xy} = ne^2/h$  is quantized ( $n$  is an integer). In real experiments at  $T \neq 0$  the  $\sigma_{xx} \neq 0$  within the plateaus in the low-field region and displays the SdH oscillations at smaller fields in which plateaus in the  $\sigma_{xy}$  are unresolved.

The purpose of this paper is to study analytically the quantum magnetic oscillations in the  $\sigma_{xx}$  in the 2D conductor with the localized states in the broaden LLs and to proof the properties (i)-(iv).

The conductivity due to the electron tunneling between the Landau orbits was calculated in [2] for the case of incoherent hopping across the layers of a layered conductor. This hopping mechanism remains in effect if Landau orbits lay within the same conducting plane or belong to the tunnel-coupled 2D conductors. The latter was proved by a recent observation of the typical IQHE behavior in the tunneling conductance of a two coupled Hall bars [3]. According to [2] the tunneling SdH conductivity can be written as a sum of the Boltzmann ( $\sigma_B$ ) and quantum ( $\sigma_Q$ ) terms:  $\sigma_{xx} = \sigma_B + \sigma_Q$ , where

$$\sigma_B = \sigma_0 \int d\varepsilon \frac{dE}{\pi} g(\varepsilon) v_x^2(\varepsilon) \left( -\frac{\partial f}{\partial E} \right) \tau S[\lambda, \delta(E, \varepsilon)], \quad (1)$$

$$\sigma_Q = \sigma_0 \int d\varepsilon \frac{dE}{\pi} g(\varepsilon) v_x^2(\varepsilon) \left( \frac{\partial f}{\partial E} \right) \frac{2\pi}{\Omega} \frac{\partial}{\partial \lambda} S[\lambda, \delta(E, \varepsilon)]. \quad (2)$$

Here  $\lambda(E) = 2\pi/\Omega\tau$ ,  $\delta(E, \varepsilon) = 2\pi(E + \varepsilon)/\hbar\Omega$ ,  $\sigma_0 = e^2 N_L/\Omega$ ,  $N_L = \Phi/S\Phi_0$  is the electron density at the LL,  $\Phi$  is the flux through a sample,  $\Phi_0 = \hbar c/2\pi e$ , and

$$S(\lambda, \delta) = \sum_{p=-\infty}^{\infty} (-1)^p e^{-|p|\lambda} \cos p\delta = \frac{\sinh \lambda}{\cosh \lambda + \cos \delta}. \quad (3)$$

The variable  $\varepsilon$  describes the LLs broadening by impurities with the density of states (DOS)  $g(\varepsilon)$ :

$$E_n(\varepsilon) = \hbar\Omega(n + 1/2) + \varepsilon. \quad (4)$$

The electron velocity  $v_x$  is related to the tunneling matrix elements by [2]

$$v_x(\varepsilon) = \frac{|t_{\varepsilon, \varepsilon}| R}{\hbar\sqrt{2}} \quad (5)$$

where  $R$  and  $\hbar\sqrt{2}/|t_{\varepsilon, \varepsilon}|$  are correspondingly the distance and the time of the tunneling. The strong point of the above equations is that we can learn much about the

$\sigma_{xx}(B, T)$  without resort to the specific models for the localization ( $B$  is the magnetic field). In any such model the  $g(\varepsilon)$  has a narrow band of delocalized states where the  $v_x(\varepsilon) \neq 0$ . It is generally accepted now that only one state, precisely at the LL ( $\varepsilon = 0$ ) is delocalized. For the localized states  $v_x(\varepsilon) = 0$ . Thus, only one level  $\varepsilon = 0$ , or a small stripe of delocalized states, contribute into Eqs.(1),(2).

The scattering time  $\tau$  in general is a model-dependent function of the energy which is inversely proportional to the scattering probability for the conducting (delocalized) electrons. The latter belong to a narrow stripe in the  $g(\varepsilon)$  while the rest of electrons are localized and produce a reservoir of states stabilizing oscillations in  $\tau$ . Besides, only  $\varepsilon = 0$  contribute into the  $\sigma_{xx}$ . Thus, we can put  $\tau = \text{const.}$  in Eqs.(1),(2) which yields:

$$\sigma_{xx} = \sigma_\tau \int \frac{dE}{\pi} \left( -\frac{\partial f}{\partial E} \right) [G_B(\lambda, E) + G_Q(\lambda, E)], \quad (6)$$

$$G_B(\lambda, E) = S[\lambda, \Delta(E)], \quad (7)$$

$$G_Q(\lambda, E) = -\lambda \frac{\partial}{\partial \lambda} S[\lambda, \Delta(E)] = -\lambda \frac{1 + \cosh \lambda \cos \Delta}{(\cosh \lambda + \cos \Delta)^2}, \quad (8)$$

where  $\Delta(E) = 2\pi E/\hbar\Omega$  and

$$\sigma_\tau = \frac{e^2 N_L \tau \langle v_x^2 \rangle}{\hbar \Omega}. \quad (9)$$

The average of the velocity squared, is given by

$$\langle v_x^2 \rangle = \frac{R^2}{2\hbar^2} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} d\varepsilon g(\varepsilon) |t_{\varepsilon, \varepsilon}|^2. \quad (10)$$

Integral in Eq.(10) is taken within the narrow stripe of the delocalized states. The functions  $G_B(\lambda, E)$  and  $G_Q(\lambda, E)$  are sharply peaked at the LLs  $E = E_n$  and between them they nearly compensate each other, as one can see in Fig.1. This important point demonstrates clearly that the Boltzmann term alone,  $G_B(\lambda, E)$ , is insufficient for the correct description and only by taking account of the quantum term,  $G_Q(\lambda, E)$ , one can explain why  $\sigma_{xx}$  tends to zero between the peaks in the IQHE. The width of the peaks in Fig.1 in the energy scale is of the order of  $\hbar/\tau$ . If the temperature  $T \gg \hbar/\tau$ , then the peaked function  $-(\partial f/\partial E)$  is broader than, the  $G_{xx}(\lambda, E) = G_B(\lambda, E) + G_Q(\lambda, E)$ , and we can approximate the  $G_{xx}(\lambda, E)$  in Eq.(6) by

$$G_{xx}(\lambda, E) \approx \frac{2}{\pi} \sum_{n=-\infty}^{\infty} \frac{\eta}{(n + 1/2 - E/\hbar\Omega)^2 + \eta^2} \quad (11)$$

where  $\eta = \lambda/2\pi$ . For  $\eta \ll 1$  Eq.(11) can be easily proved analytically with the help of the identity [2]

$$\frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{\eta}{(n + a)^2 + \eta^2} = \frac{\sinh 2\pi\eta}{\cosh 2\pi\eta - \cos 2\pi a}. \quad (12)$$

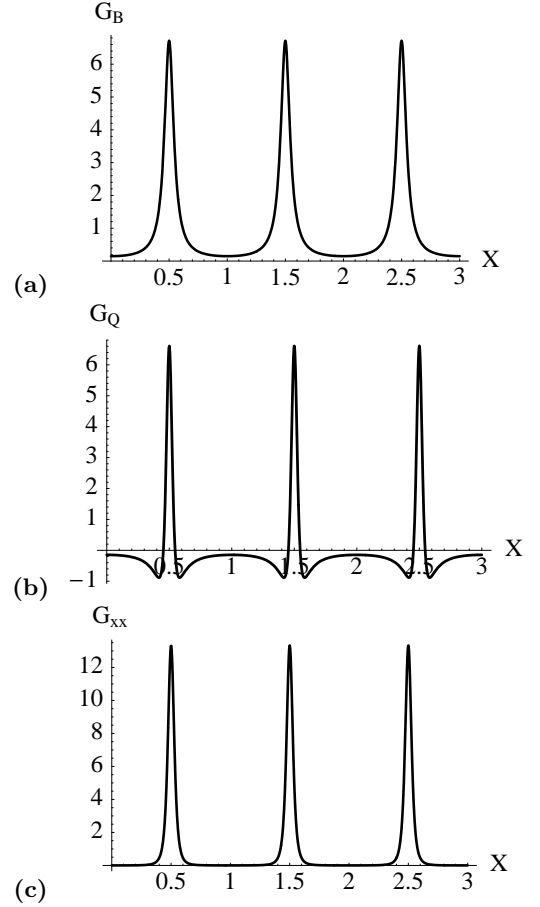


FIG. 1: The Boltzmann,  $G_B = S(\lambda, 2\pi X)$  (Fig.1a), and the quantum,  $G_Q = -\lambda \frac{\partial}{\partial \lambda} S(\lambda, 2\pi X)$  (Fig.1b), contributions into the conductivity  $\sigma_{xx}$  in Eq.(6), and their sum  $G_{xx} = G_B + G_Q$  (Fig.1c).  $X = E/\hbar\Omega$ ,  $\lambda = 0.3$ .

Thus, for high temperatures,  $T \gg \hbar/\tau$ , we have

$$\sigma_{xx}(B) \approx \sigma_\tau \frac{\hbar\Omega}{4\pi T} \sum_n \cosh^{-2} \left( \frac{E_n - \mu}{T} \right). \quad (13)$$

This sharply-peaked function of the  $\hbar\Omega$  is shown in Fig.2. The same function describes the quantum magnetic oscillations of the ultrasound absorption in metals [4]. A temperature dependence of the peaks in  $\sigma_{xx}(B)$  for different temperatures is plotted in Fig.3. Under the condition  $\hbar\Omega/T \gg 1$ , the conductivity  $\sigma_{xx}$  at the maxima (i.e. when  $E_n = \mu$ ) is given by  $\sigma_{xx} = \sigma_\tau \frac{\hbar\Omega}{4\pi T}$ . At the minima (i.e. when the chemical potential  $\mu$  falls between the LL) the conductivity  $\sigma_{xx}$  is exponentially small:  $\sigma_{xx} = \sigma_\tau \frac{\hbar\Omega}{4\pi T} \exp(-\frac{\hbar\Omega - E_0}{T})$  ( $E_0$  is a position of the  $\mu$  between the LL). Such an activation dependence is well established in the  $\sigma_{xx}(T)$  in the IQHE regime [1]. At lower temperatures,  $T \ll \hbar/\tau$ , one can approximate the  $(-\partial f/\partial E)$  by  $\delta(E - \mu)$ , to obtain

$$\sigma_{xx} \approx \sigma_\tau (G_B[\lambda, \Delta(\mu)] + [G_Q\lambda, \Delta(\mu)]). \quad (14)$$

The  $\sigma_{xx}$  in (14) is sharply peaked function of the  $\Delta(\mu) =$

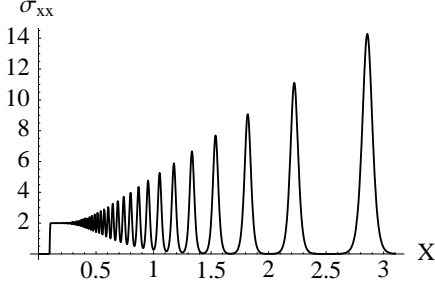


FIG. 2: The conductivity  $\sigma_{xx}$  [see Eq.(13)] in units of  $\sigma_\tau$  as a function of the  $X = \hbar\Omega$ . The conventional energy units accepted in which  $T = 0.2$  and  $E_F = 10$ .

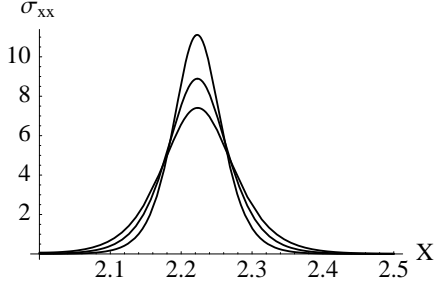


FIG. 3: The same as in Fig.2 for three different temperatures  $T = 0.2$ ,  $T = 0.25$ , and  $T = 0.3$  (from top to bottom).

$2\pi\mu/\hbar\Omega$  as shown in Fig.1. The Boltzmann and the quantum terms in Eq.(14) nearly compensate each other between the peaks which in the limit  $\eta \rightarrow 0$  become narrow Lorentzians of Eq.(11). The temperature dependence of the  $\sigma_{xx}$  at  $T \ll \hbar/\tau$  in Eq.(14) comes only from the  $\sigma_\tau$  due to the VRH mechanism. The VRH concept in the IQHE problem is now well established [5, 6]. It was introduced in [7] and well describes the scaling properties of the peaks in the  $\sigma_{xx}$  within the plateau-to-plateau transition region. The diagonal and the Hall conductivities in this region are related by the "semicircle" law [8]. In samples with the mobility of the order  $10^2$  Vs at a few tens mK the best experimental fit yields [5, 6]:

$$\sigma_\tau = \frac{A}{T} \exp(-\sqrt{T_0/T}), \quad T_0 = C \frac{e^2}{\epsilon\xi}. \quad (15)$$

The characteristic temperature  $T_0$  is proportional to the Coulomb energy at the localization length  $\xi(\nu)$  and  $\epsilon$  is the dielectric constant,  $C \sim 1$ . Many experiments and numerical calculations testify in favor of a universal critical behavior of the localization length  $\xi(\nu) \propto |\nu - \nu_c|^{-\gamma}$  near the Landau levels [1, 5, 6, 7]. Here  $\nu = N\Phi_0/B$  is the filling factor,  $\nu_c$  is the critical filling factor, and  $\gamma \approx 2.35$  is a universal critical exponent. The divergency of the  $\xi(\nu)$  at  $\nu_c$  means that this is a critical point for the transition from the dielectric to the conducting state.

Eq.(15) directly follows from Eqs.(9), and (10). In the spirit of the VRH approach, we can estimate the  $|t_{\epsilon,\epsilon}|^2$

as proportional to the the electron hopping probability between the two 1D closed equipotential impurity-potential-contours at which Landau orbitals are localized. If  $R$  is a distance of the hopping, then

$$|t_{\epsilon,\epsilon}|^2 \propto \exp \left[ - \left( \frac{1}{RN(0)T} + \frac{2R}{\xi} \right) \right]. \quad (16)$$

Here we take account of the thermal activation which helps the tunneling if the initial and final levels are within the energy stripe of the order of  $1/RN(0)$ , where  $N(0)$  is the DOS at the Fermi level. Thus, the optimal hopping distance is  $R = \sqrt{\xi/2N(0)T}$ . Putting this value into Eqs.(10),(16) we have  $\langle v_x^2 \rangle \propto 1/T \exp(-\sqrt{T_0/T})$  which, in view of Eq.(9) result in the VRH conductivity given by Eq.(15). The VRH concept was originally applied to the problem of the conductivity peak broadening  $\Delta\nu$  in [7]. It was shown that the temperature, current, and frequency dependencies of the  $\Delta\nu$  can be well described within this paradigm. Here we derived a prefactor  $A/T$  which also have been observed in the conductivity  $\sigma_{xx}(T)$  [5, 6]. However, it should be noted that the prefactor  $A/T$  is absent in the experiments in which a conductance was measured [9, 10]. The difference is because the conductivity in Eq.(1),(2) is proportional to the  $v_x^2 \propto R^2 \propto 1/T$ . The conductance  $\sigma_{xx}^c(T) \propto (e^2/\hbar)|t_{\epsilon,\epsilon}|^2$  and does not contain a factor  $R^2 \propto 1/T$ . Therefore, at the same conditions as in Eq.(14) the conductance is:

$$\sigma_{xx}^c \approx \sigma_\tau^c (G_B[\lambda, \Delta(\mu)] + G_Q[\lambda, \Delta(\mu)]), \quad (17)$$

$$\sigma_\tau^c = A_c \exp(-\sqrt{T_0/T}). \quad (18)$$

Since  $T_0 \propto 1/\xi \propto |\nu - \nu_c|^\gamma$  the function  $\sigma_\tau^c(\nu)$  has a fixed maximum value  $\sigma_\tau^c = A_c$  at  $\nu = \nu_c$  for different temperatures. This remarkable property of the conductance is firmly established in the VRH regime at low temperatures [8, 9, 10]. So far we assumed that the chemical potential is a constant. In 2D conductors  $\mu(B)$  is an oscillating function [11] satisfying the equation [2]:

$$\mu = E_f \pm \frac{\hbar\Omega}{\pi} \arctan \left[ \frac{\sin(2\pi\mu/\hbar\Omega)}{e^\nu + \cos(2\pi\mu/\hbar\Omega)} \right]. \quad (19)$$

The sign (-) here stands for the direct and (+) for the inverse sawtooth. The amplitude of these oscillations is of the order of the  $\hbar\Omega$  which is small compared to the  $E_F$ . It was shown in [2] that in a quasi 2D layered conductor the peaks in the magnetic conductivity across the layers are split in the case  $\mu(B)$  is an inverse sawtooth function. The very same effect holds for the  $\sigma_{xx}$ , as shown in Fig.4, which displays the  $\sigma_{xx}(B)$  according to Eq.(14) with the  $\mu(B)$  given by Eq.(19). We also take account of the spin-splitting which is easy to incorporate by the substitution  $\mu \rightarrow \mu \pm \mu_e B$  into the right-hand-side of Eq.(19) and by average it over two spin configurations ( $\mu_e$  is the

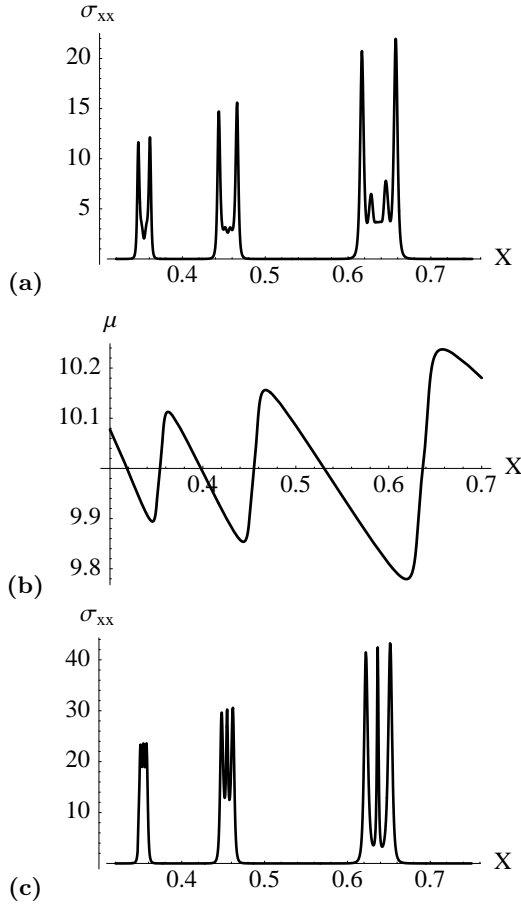


FIG. 4: The conductivity  $\sigma_{xx}(X)$  given by Eq.(14) (Fig.4a) in which the chemical potential  $\mu(X)$  oscillates as in Fig.4b (a direct sawtooth, see Eq.(17) and text). The choice of the units is the same as in Fig.2. The spin-splitting parameter  $s = 0.093$ ,  $E_F = 10$ ,  $\nu = 0.06$ , and  $X = \hbar\Omega$ . In Fig.4c  $s = 0$ .

magnetic moment of electron). The spin-splitting parameter  $s = 2\pi\mu_e B/\hbar\Omega$  can be rewritten in terms of the  $g$ -factor and the effective mass to the electron mass ratio  $s = \pi g(m^*/m)$ . In GaAs  $g \approx 0.44$  and  $(m^*/m) \approx 0.068$  which yield  $s \approx 0.093$ . This value gives a pronounced splitting in the peaks in Fig.4a, but it is much less noticeable in Fig.4b for  $\mu(B)$ . The shape of peaks in the absence of splitting ( $s = 0$ ) is shown in Fig.4c. For the direct sawtooth shape of the  $\mu(B)$  the peak-splitting is shown in Fig.5 in a more detail. As explained in [2] the difference in the shape of the  $\sigma_{xx}(B)$  is because the equation for the split-peaks positions has different number of the real and imaginary roots for the direct and inverse shapes of the sawtooth function  $\mu(B)$ .

In conclusion, the considered model of the hopping conductivity describes the different regimes in the diagonal conductivity  $\sigma_{xx}$ , as stated above in (i)-(iv). It also explains why the VRH exponent in the conductivity  $\sigma_{xx}$  corresponds to a 1D system while the system in question is a 2D. It is worth to note that the peak-split shape

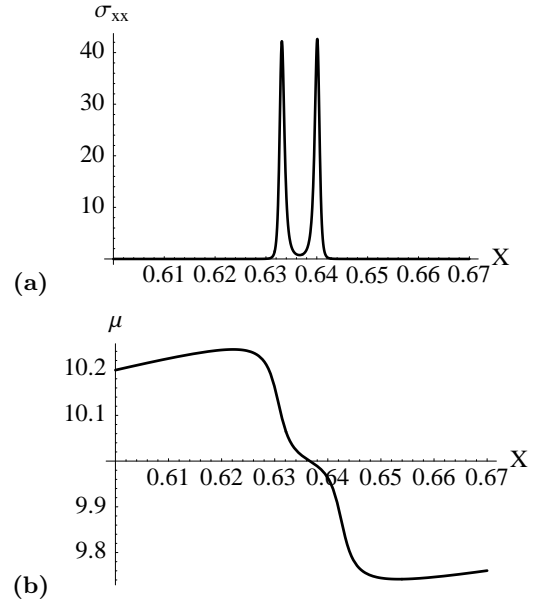


FIG. 5:  $\sigma_{xx}(X)$  and  $\mu(X)$  - direct sawtooth,  $\nu = 0.03$ .

in Fig.4(a) is typical for the IQHE conductors with the high mobility of electrons. The approach developed is open for the usage of specific models for the localization (see Eq.(10)). The role of the quantum term in the  $\sigma_{xx}$  is similar to that considered in [2, 12, 13] for the conductivity across the layers in organic conductors.

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